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The littlest Higgs

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Abstract: We present an economical theory of natural electroweak symmetry breaking, generalizing an approach based on deconstruction. This theory is the smallest extension of the Standard Model to date that stabilizes the electroweak scale with a naturally light Higgs and weakly coupled new physics at TeV energies. The Higgs is one of a set of pseudo Goldstone bosons in an $\mathrm{SU}(5)/\mathrm{SO}(5)$ nonlinear sigma model. The symmetry breaking scale f is around a TeV, with the cutoff $\Lambda \lesssim 4\pi f \sim 10\,\mathrm{TeV}$. A single electroweak doublet, the "little Higgs", is automatically much lighter than the other pseudo Goldstone bosons. The quartic self-coupling for the little Higgs is generated by the gauge and Yukawa interactions with a natural size $\mathcal{O}(g^2, \lambda_t^2)$, while the top Yukawa coupling generates a negative mass squared triggering electroweak symmetry breaking. Beneath the TeV scale the effective theory is simply the minimal Standard Model. The new particle content at TeV energies consists of one set of spin one bosons with the same quantum numbers as the electroweak gauge bosons, an electroweak singlet quark with charge 2/3, and an electroweak triplet scalar. One loop quadratically divergent corrections to the Higgs mass are cancelled by interactions with these additional particles.

Keywords: Beyond Standard Model, Higgs Physics.

Contents

1. Introduction

The Standard Model provides an excellent effective field theory description of almost all particle physics experiments. But at what scale Λ does this effective description break down? In the Standard Model the electroweak symmetry breaking scale, of order M_W, is set by the Higgs mass parameter m_H^2. There is no symmetry reason why m_H^2/Λ^2 should be small, and since this ratio receives quantum corrections of order α_W/π, a reasonable expectation is that $\Lambda \lesssim$ a few TeV [1]. Consequently we expect new physics at the TeV scale that protects the electroweak scale from large radiative corrections. Candidates for this physics include technicolor, a low fundamental quantum gravity scale and supersymmetry. Supersymmetry is especially popular as it provides a weakly coupled description of TeV scale physics in addition to stabilizing a light Higgs, as seems favored by precision electroweak data. While these options have been vigorously explored over the past several decades, none have yet been confirmed experimentally. Consequently it is important to explore qualitatively new ideas for natural electroweak symmetry breaking, especially those that are weakly coupled and stabilize a light Higgs.

Why should the Higgs be light? An early dream [2, 3], dating from the seventies, was the construction of the Higgs as a pseudo-Goldstone boson. The first realistic attempt along these lines was made in the eighties: the Georgi-Kaplan Composite Higgs [4]–[8]. In these models the Higgs is the pseudo-Goldstone boson of a nonlinearly realized approximate

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global symmetry, analogous to the pions and kaons. Provided that f, the analog of the pion decay constant, is much larger than the electroweak symmetry breaking scale, the minimal Standard Model with a light Higgs is a good effective description of electroweak symmetry breaking. Unfortunately in the Georgi-Kaplan model a hierarchy between f and the electroweak scale M_W is possible only by fine tuning parameters, and so this theory does not provide a natural stabilization of the weak scale.

In the last year, inspired by "dimensional deconstruction" [12, 13], the Higgs as a pseudo-Goldstone boson has been successfully realized [14, 15]. These models are characterized by a "theory space", summarizing the gauge and global symmetries of the theory. The physics is described by a nonlinear sigma model in which a special subset of the pseudo-Goldstone bosons are naturally light, despite having gauge, Yukawa, and self-interactions of order one [14].

These models have a number of global symmetries, any one of which is sufficient to ensure a massless Higgs. These symmetries are only approximate, explicitly broken by gauge, Yukawa and scalar couplings. However each such coupling alone preserves enough of the global symmetry to forbid a Higgs mass. Quadratically divergent corrections to the Higgs mass arise only at multi-loop order, when several such interactions act in concert, making the small Higgs mass natural. Such exceptionally light pseudo-Goldstone bosons were dubbed "little Higgses" [15]. In a companion paper [16] we present the simplest theory space describing a little Higgs that naturally triggers electroweak symmetry breaking.

In this paper we show that the little Higgs phenomenon is independent of theory space, arising in nonlinear sigma models with no obvious nontrivial theory space description. This allows us to realize a little Higgs in the simplest possible way, arising from an SU(5)/ SO(5) nonlinear sigma model. The spectrum below a TeV is as economical as possible, consisting of only the minimal Standard Model with a single light Higgs. This Higgs is a pseudo-Goldstone boson whose decay constant f is of order a TeV. At the TeV scale a small number of additional scalars, vector bosons and quarks cancel the one loop quadratic divergence in the Higgs mass without fine tuning or supersymmetry. These additional degrees of freedom represent the smallest extension of the Standard Model to date stabilizing the weak scale with a light Higgs and perturbative physics at TeV energies. The cutoff of this theory can be as high as $4\pi f \sim 10$ TeV, where the nonlinear sigma model becomes strongly coupled. The $\mathrm{SU}(5) \rightarrow \mathrm{SO}(5)$ symmetry breaking can easily arise from fermion condensation through technicolor-like strong interactions [8]. However, unlike technicolor, the new strong interactions need not appear until a scale of order 10 TeV, with small impact on precision electroweak parameters.

2. Requirements

Our goal is to realize the Higgs as the pseudo-Goldstone boson of a broken symmetry in a way which ensures that its mass is not quadratically sensitive to the cutoff at one loop order. This little Higgs will then be weakly coupled up to energies one loop factor above the weak scale, around 10 TeV.

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We begin by assuming that the Higgs is part of a pseudo-Goldstone multiplet parameterizing a coset space G/H, with the decay constant, f, on the order of a TeV. For phenomenological purposes, the origin of this symmetry breaking pattern is irrelevant at energies below the cutoff scale $\Lambda \sim 4\pi f$ where additional UV physics must enter. The sigma model does not Higgs the electroweak group at the scale Λ, so the subgroup H should contain $SU(2) \times U(1)$. As in the Standard Model, the electroweak gauge interactions will naïvely induce a one loop quadratically divergent mass for the Higgs doublet. To avoid this we use a mechanism familiar from models of deconstruction: We assume that G contains a weakly gauged subgroup consisting of *two copies* of $SU(2) \times U(1)$: $G \supset G_1 \times G_2 = [SU(2) \times U(1)]^2$. Each of the G_i must commute with a different subgroup of G that acts non-linearly on the Higgs. The combination of both weak gauge interactions breaks all global symmetries which act on the Higgs, and when both are included the Higgs ceases to be an exact Goldstone boson. The quadratically divergent contributions to the Higgs mass from gauge interactions must involve both couplings, and hence first appear at two loops. In this case the Higgs mass squared is radiatively stable with a cutoff of order 10 TeV .

Let us now look for a simple implementation of the above requirements. Since G contains $[SU(2) \times U(1)]^2$ it must be at least rank 4. Also G must contain two different subgroups, each of the form $G_i \times X_i$, $i = 1, 2$. Furthermore each X_i must contain an $SU(2) \times U(1)$ subgroup with some X_i generators transforming like doublets. For instance, we might (and will) take $X_i = SU(3)_i$ and $G = SU(5)$. The doublet generators of the X_i should not lie entirely in H. Assuming $G = SU(5)$, an obvious candidate for H is $SO(5)$, which contains the diagonal sum of G_1 and G_2. With the symmetry breaking pattern $SU(5) \to SO(5)$, the 14 Goldstone bosons decompose under the electroweak $SU(2) \times U(1)$ as

$$\mathbf{1}_0 \oplus \mathbf{3}_0 \oplus \mathbf{2}_{\pm 1/2} \oplus \mathbf{3}_{\pm 1} . \tag{2.1}$$

The first two sets of bosons are removed by the Higgs mechanism when $G_1 \times G_2$ breaks to the electroweak group. The next set are the little Higgs and its hermitian conjugate, and the last set is an additional complex triplet. We shall see that the triplet acquires a TeV scale mass at one loop from gauge interactions. The Higgs quartic self-coupling arises from integrating out this massive triplet. Thus the triplet coupling to the little Higgs naturally cancels the one loop quadratic divergence in the little Higgs mass from the Higgs self coupling.

Each of the G_i gauge groups commutes with a different $SU(3)$ global symmetry subgroup of $SU(5)$. Examining one of these $SU(3) \times SU(2) \times U(1)$ global-local product subgroups, we see that the first three sets of Goldstone fields above (including the little Higgs) transform non-linearly under the $SU(3)$. Thus neither of the G_i alone can generate a potential for the Higgs. The two gauge groups together however completely break all global symmetry protecting the Higgs. Therefore Higgs potential terms must involve both gauge couplings and a UV sensitive Higgs mass cannot be generated at one loop. The triplet mass is not protected by any global symmetry and indeed receives a quadratically cutoff sensitive mass from the gauge interactions at one loop. Hence below a TeV the sigma model contains a single Higgs doublet and nothing else. At the TeV scale there is an additional triplet scalar, and four gauge bosons: an electroweak triplet $W'^{\pm 0}$, and a neutral electroweak singlet B'^0.

3. The model

Our minimal theory is based on an SU(5)/ SO(5) non-linear sigma model, the same structure considered in the original Composite Higgs models. Since this non-linear sigma model may not be as familiar as the QCD chiral lagrangian for pions, we describe it in some detail here. The breaking of SU(5) $\to$ SO(5) guarantees 14 Goldstone bosons. In order to construct the non-linear sigma model, it is convenient to imagine for a moment that this breaking arises from a vacuum expectation value for a 5×5 symmetric matrix Φ, which transforms as $\Phi \to V \Phi V^T$ under SU(5). A vacuum expectation value for Φ proportional to the unit matrix then breaks SU(5) $\to$ SO(5). For later convenience, we use an equivalent basis where the vacuum expectation value for the symmetric tensor points in the Σ_0 direction where Σ_0 is

$$\Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix} . \tag{3.1}$$

The unbroken SO(5) generators satisfy

$$T_a \Sigma_0 + \Sigma_0 T_a^T = 0 \tag{3.2}$$

while the broken generators obey

$$X_a \Sigma_0 - \Sigma_0 X_a^T = 0 \,. \tag{3.3}$$

As usual, the Goldstone bosons are fluctuations about this background in the broken directions $\Pi \equiv \pi^a X^a$, and can be parameterized by the non-linear sigma model field

$$\Sigma(x) = e^{i\Pi/f} \Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f} \Sigma_0 \,, \tag{3.4}$$

where the last step follows from (3.3).

We now introduce the gauge and Yukawa interactions which explicitly break the global symmetry. As stressed in the previous section, these are chosen to ensure an enhanced SU(3) global symmetry in the limit where any of the couplings are turned off. We begin by gauging a $G_1 \times G_2 = [\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ subgroup of the SU(5) global symmetry. The generators of the first $G_1 = \mathrm{SU}(2) \times \mathrm{U}(1)$ are embedded into SU(5) as

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & & \\ & & \\ & & \end{pmatrix} , \qquad Y_1 = \mathrm{diag}(-3,-3,2,2,2)/10 \tag{3.5}$$

while the generators of the second $\mathrm{SU}(2) \times \mathrm{U}(1)$ are given by

$$Q_2^a = \begin{pmatrix} & & \\ & & \\ & & -\sigma^{a*}/2 \end{pmatrix} , \qquad Y_2 = \mathrm{diag}(-2,-2,-2,3,3)/10 \,. \tag{3.6}$$

In the next section, we will see that the $G_1 \times G_2$ gauge symmetry is broken to the diagonal $\mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup which we identify with the electroweak gauge symmetry. It is therefore convenient to write the Goldstone boson matrix Π in terms of fields with definite electroweak quantum numbers

$$\Pi = \begin{pmatrix} & \frac{h^\dagger}{\sqrt{2}} & \phi^\dagger \\ \frac{h}{\sqrt{2}} & & \frac{h^*}{\sqrt{2}} \\ \phi & \frac{h^T}{\sqrt{2}} & \end{pmatrix} \tag{3.7}$$

where h is the Higgs doublet, $h = (h^+, h^0)$ and ϕ represents the triplet as a symmetric two by two matrix, which transforms as a $\mathbf{3}_1$ under the electroweak group. We have ignored the Goldstone bosons that are eaten in the Higgsing of $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2 \to \mathrm{SU}(2) \times \mathrm{U}(1)$.

These gauge interactions satisfy the requirements of the previous section. When the couplings of G_1 are turned off, there is an enhanced $\mathrm{SU}(3)_1$ global symmetry living in the upper 3×3 block of $\mathrm{SU}(5)$; when the gauge interactions of G_2 are turned off there is an $\mathrm{SU}(3)_2$ global symmetry living in the lower 3×3 block.

The tree-level lagrangian for the model is given by

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_t + \mathcal{L}_\psi \tag{3.8}$$

Here $\mathcal{L}_K$ contains the kinetic terms for all the fields; $\mathcal{L}_t$ generates the top Yukawa coupling; and $\mathcal{L}_\psi$ generates the remaining small Yukawa couplings. In detail, $\mathcal{L}_K$ includes the conventional kinetic terms for the gauge fields and Fermions, as well as the leading two-derivative term for the non-linear sigma model

$$\frac{f^2}{4} \operatorname{tr} |D_\mu \Sigma|^2 \tag{3.9}$$

where the covariant derivative of Σ is given by

$$D\Sigma = \partial\Sigma - \sum_j \left\{ i g_j W_j^a (Q_j^a \Sigma + \Sigma Q_j^{aT}) + i g'_j B_j (Y_j \Sigma + \Sigma Y_j^T) \right\}. \tag{3.10}$$

The g_i, g'_i are the couplings of the $[\mathrm{SU}(2) \times \mathrm{U}(1)]_i$ groups. In order to introduce a large top Yukawa coupling while avoiding the associated large quadratic divergence in the Higgs mass, we add a pair of colored Weyl Fermions $\tilde{t}, \tilde{t}^c$ in addition to the usual third-family weak doublet $q_3 = (t_3, b_3)$ and weak singlet ${u'_3}^c$. It is convenient to group the doublet together with $\tilde{t}$ into a row vector $\chi = (b_3\, t_3\, \tilde{t})$. $\mathcal{L}_t$ is given by

$$\mathcal{L}_t = \lambda_1 f \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} {u'_3}^c + \lambda_2 f \tilde{t} \tilde{t}^c + \text{h.c.}, \tag{3.11}$$

where the indices i, j, k are summed over $1, 2, 3$ and x, y are summed over $4, 5$. This interaction fulfills our requirements: the λ_1 interaction preserves the $\mathrm{SU}(3)_1$ and breaks $\mathrm{SU}(3)_2$, while λ_2 does the converse. To see that $\mathcal{L}_t$ generates a top Yukawa coupling we expand $\mathcal{L}_t$ to first order in the Higgs h:

$$\mathcal{L}_t = \lambda_1 (q_3 h + f\tilde{t}) {u'_3}^c + \lambda_2 f \tilde{t} \tilde{t}^c + \cdots. \tag{3.12}$$

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Clearly $\tilde{t}$ marries one linear combination of ${u'_3}^c$ and $\tilde{t}^c$ to become massive. Integrating out this heavy quark, the remaining combination u_3^c has the desired Yukawa coupling to q_3

$$\lambda_t\, q_3 h\, u_3^c\,, \qquad \text{where} \qquad \lambda_t = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}\,. \tag{3.13}$$

The mixing of the top quark with vector-like Fermions at the TeV scale is similar to Frogatt-Nielsen models of flavor [9] and the top see-saw [10, 11]. Finally, the interactions in $\mathcal{L}_\psi$ encode the remaining Yukawa couplings of the Standard Model. These couplings are small so that the 1-loop quadratically divergent contributions to the Higgs mass they induce are negligible with a cutoff $\Lambda \sim 10\,\text{TeV}$. For the up sector we can take $\mathcal{L}_\psi$ to have exactly the same form as $\mathcal{L}_t$, except that the $\tilde{t}, \tilde{t}^c$ fields are unnecessary. For the down and charged lepton sector we use the same lagrangian with Σ replaced by Σ^*.

The U(1) charges of the Fermions are chosen to ensure gauge invariance. As we will see in the next section, the $G_1 \times G_2$ symmetry is Higgsed to the diagonal Standard Model SU(2)×U(1) gauge group, so the U(1) charges must be chosen to yield the correct Standard Model quantum numbers. We do not concern ourselves with cancellation of the $G_1 \times G_2$ anomalies in this low energy effective theory, since there may be additional Fermions at the cutoff which cancel the anomalies involving the broken subgroup. We insist only that Standard Model anomalies cancel, since fermions in a chiral representation of the Standard Model can have only weak scale masses. Notice also that our two U(1) generators Y_1, Y_2 are not orthogonal, but we have not included any kinetic mixing term between them. Any mixing between the U(1)'s in our effective theory arises at loop level and is sufficiently small. In this model our choice for the U(1)'s is dictated by the requirement that in the limit where any one of the U(1)'s is turned off there is an SU(3) global symmetry. If we demand that the two U(1)'s *both* fit inside the SU(5) global symmetry, the Y_1, Y_2 are fixed. It is easy to add new U(1) factors that commute with the SU(5) global symmetry which allow Y_1, Y_2 to be chosen orthogonal. Alternatively, enlarging to an SU(N)/SO(N) non-linear sigma model allows sufficient room to embed two orthogonal U(1)'s in the SU(N) global symmetry while still satisfying our requirements.

4. Radiative corrections and electroweak symmetry breaking

Since the gauge and Yukawa interactions explicitly break the global SU(5) symmetry, these interactions will select a preferred alignment for Σ. To find the vacuum, we compute the 1-loop radiative potential for Σ from the gauge, scalar and Yukawa sectors.

First consider the gauge sector. The largest corrections come from 1-loop gauge quadratic divergences, which are easily extracted from the quadratically divergent part of the Coleman-Weinberg potential [17]:

$$\frac{\Lambda^2}{16\pi^2}\,\mathrm{tr}\, M_V^2(\Sigma) \tag{4.1}$$

where $M^2(\Sigma)$ is the gauge boson mass matrix in a background Σ. $M_V^2(\Sigma)$ can be easily read off from the covariant derivative for Σ (3.10), giving a potential

$$cg_j^2 f^4 \sum_a \operatorname{tr}\left[(Q_j^a\Sigma)(Q_j^a\Sigma)^*\right] + c{g'_j}^2 f^4 \operatorname{tr}\left[(Y_j\Sigma)(Y_j\Sigma)^*\right] \tag{4.2}$$

Here we have used $\Lambda \sim 4\pi f$, and c is an $\mathcal{O}(1)$ constant whose precise value is sensitive to the UV physics at the scale Λ. The presence of this quadratic divergence implies that the lagrangian (3.8) is incomplete: we must include all operators consistent with the symmetries of the theory with natural sizes determined by naïve dimensional analysis [18, 19, 20]. At second order in the gauge couplings and momenta (4.2) is the unique gauge invariant term transforming properly under the global SU(5) symmetry. This potential is analogous to that generated by electromagnetic interactions in the pion chiral lagrangian, which shift the masses of $\pi^\pm$ from that of the π^0 [21]. For the pion masses, naïve dimensional analysis works beautifully. Moreover, in analogy to the chiral lagrangian, we assume that c is positive. This implies that the vacuum is correctly aligned, the electroweak group remains unbroken by the sigma model, and the triplet has a positive TeV sized mass $m_\phi \sim gf$.

To find the Higgs potential we expand the lagrangian (4.2) in the pseudo-Goldstone fields. The form of the potential is determined by the global symmetry transformation properties of the Higgs and triplet fields. The G_1 gauge interactions leave the $\mathrm{SU}(3)_1$ symmetry invariant, part of which acts on the Higgs and triplet fields as

$$h_i \to h_i + \epsilon_i + \cdots \tag{4.3}$$

$$\phi_{ij} \to \phi_{ij} - i(\epsilon_i h_j + \epsilon_j h_i) + \cdots \tag{4.4}$$

while G_2 leaves $\mathrm{SU}(3)_2$ symmetry invariant, and acts as

$$h_i \to h_i + \eta_i + \cdots \tag{4.5}$$

$$\phi_{ij} \to \phi_{ij} + i(\eta_i h_j + \eta_j h_i) + \cdots . \tag{4.6}$$

Hence to quadratic order in ϕ and quartic order in h the potential is

$$c(g_1^2 + {g'_1}^2)f^2 \left|\phi_{ij} + \frac{i}{2f}(h_i h_j + h_j h_i)\right|^2 + c(g_2^2 + {g'_2}^2)f^2 \left|\phi_{ij} - \frac{i}{2f}(h_i h_j + h_j h_i)\right|^2 \tag{4.7}$$

As previously claimed the gauge interactions induce a mass for the triplet of order gf, while the little Higgs remains massless.

There is also a quadratically divergent Coleman-Weinberg potential generated by the Fermion loop, which requires the inclusion of the operator

$$-c'\lambda_1^2 \epsilon^{wx}\epsilon_{yz}\epsilon^{ijk}\epsilon_{kmn}\Sigma_{iw}\Sigma_{jx}\Sigma^{*my}\Sigma^{*nz} + \text{h.c.} \tag{4.8}$$

This operator is $\mathrm{SU}(3)_1$ symmetric, and therefore generates a potential of the same form as the first term in (4.7), with coefficient $-c'\lambda_1^2$. As long as $c(g_1^2 + {g'_1}^2 + g_2^2 + {g'_2}^2) - c'\lambda_1^2 > 0$, the triplet mass squared remains positive. At energies beneath the triplet mass, we can integrate this particle out and get a quartic potential for the Higgs

$$\lambda(h^\dagger h)^2\,, \qquad \text{where} \quad \lambda = c\frac{(g_1^2 + {g'_1}^2 - c'/c\lambda_1^2)(g_2^2 + {g'_2}^2)}{g_1^2 + {g'_1}^2 - c'/c\lambda_1^2 + g_2^2 + {g'_2}^2} \tag{4.9}$$

As advertised the interactions combine to give the Higgs a quartic potential determined by gauge and Yukawa couplings, and no mass term.

The remaining part of the vector boson contribution to the Coleman-Weinberg potential is

$$\frac{3}{64\pi^2}\,\mathrm{tr}\,M_V^4(\Sigma)\log\frac{M_V^2(\Sigma)}{\Lambda^2}\,. \tag{4.10}$$

This gives a logarithmically enhanced positive Higgs mass squared

$$\frac{3}{64\pi^2}\left\{3g^2 {M'_W}^2\log\frac{\Lambda^2}{{M'_W}^2}+g'^2{M'_B}^2\log\frac{\Lambda^2}{{M'_B}^2}\right\} \tag{4.11}$$

where M'_W is the mass of the heavy SU(2) triplet of gauge bosons and M'_B is the mass of the heavy U(1) gauge boson.

There is a similar Coleman-Weinberg potential from the scalar self-interactions in (4.2) and in (4.8) which also give logarithmically enhanced positive contributions to the Higgs mass squared:

$$\frac{\lambda}{16\pi^2}M_\phi^2\log\frac{\Lambda^2}{M_\phi^2} \tag{4.12}$$

where M_ϕ is the triplet scalar mass.

The remaining part of the Fermion loop contribution to the Coleman-Weinberg potential is

$$-\frac{3}{16\pi^2}\,\mathrm{tr}\left(M_f(\Sigma)M_f^\dagger(\Sigma)\right)^2\log\frac{M_f(\Sigma)M_f^\dagger(\Sigma)}{\Lambda^2} \tag{4.13}$$

where $M_f(\Sigma)$ is the fermion mass matrix in a background Σ. This potential gives a logarithmically enhanced, *negative* contribution to the Higgs mass squared

$$-\frac{3\lambda_t^2}{8\pi^2}m'^2\log\frac{\Lambda^2}{m'^2} \tag{4.14}$$

where m' is the mass of the heavy fermion. This can dominate over the positive gauge and scalar contributions, triggering electroweak symmetry breaking.

What is the mass of the physical Higgs in this model? The Higgs mass is determined by the Higgs quartic coupling λ, which receives significant contributions from the gauge interactions (4.9) and from the operator (4.8). Both of these contributions are proportional to unknown coefficients c, c' of order one, encoding information about the UV physics. We can obtain a more predictive theory for the Higgs mass through an alternative model for the top Yukawa coupling. We introduce fermions in complete SU(5) multiplets, transforming as $(5,3)$ and $(5,\bar{3})$ under $\mathrm{SU}(5)\times\mathrm{SU}(3)_{\mathrm{color}}$ and coupling to the Σ field in an SU(5) symmetric fashion. Such multiplets might be expected in strongly coupled theories. The left handed top and bottom are a mixture of a component of a $(5,3)$ multiplet and an additional quark doublet field $q\sim(t,b)$ and the anti-top is a similar mixture of a component of the $(5,\bar{3})$ field and an SU(5) singlet field t^c. We break the SU(5) symmetry only through explicit fermion mass terms connecting the q and t^c to the SU(5) multiplet fermions with the appropriate quantum numbers. This form of symmetry breaking is soft enough to not induce quadratic divergences at one loop, and so the gauge contribution dominates the Higgs quartic potential. In this case the Higgs mass is parametrically of order the Z mass, $m_H \sim M_Z$!

5. Precision electroweak tests

New physics which couples to the Higgs and gauge bosons is constrained by precision electroweak measurements, which agree well with the predictions of the minimal Standard Model. The contributions of new, weakly coupled particles are generally suppressed by factors of $M_W^2/M_{\rm new}^2$, unless the mass $M_{\rm new}$ of the new particles is due to electroweak symmetry breaking, in which case the contributions of heavy particles to low energy physics do not decouple and can be large, as occurs in technicolor theories [22, 23, 24]. In our case all the new couplings are weak and all the new particle masses are symmetric under the electroweak interactions, hence their loop contributions are of order $(g^2/(16\pi^2))M_W^2/(gf)^2$, i.e. of similar size to 2-loop Standard Model corrections. Corrections of this size are typically smaller than the current experimental uncertainties. The main effects on low energy observables will arise from tree-level exchange of heavy particles, which give effects comparable to Standard Model loops. For instance the ϕ scalar will have a custodial SU(2) breaking trilinear coupling with the Higgs of the form $h\phi h$. Integrating out the ϕ at tree level will induce a custodial SU(2) violating dimension six operator, which contributes to the T (or ρ) parameter. Fortunately, the coefficient of the trilinear coupling is not expected to be large. The gauge contribution is proportional to $g_1^2 - g_2^2 + {g'_1}^2 - {g'_2}^2$ while the Fermion contribution is proportional to $-c'\lambda_1^2$. This would be absent in the complete SU(5) multiplet model of the top mass. Another source of tree level corrections is due to the exchange of the heavy gauge bosons, which have tree-level couplings to Fermions. For instance, W' exchange can contribute to muon decay and affect the muon lifetime. Similar studies of the contributions of heavy gauge bosons to precision electroweak corrections give lower bounds on the masses as tight as 3 TeV [25]. The bounds would be weaker in the present case if, for instance, the two SU(2)'s have unequal couplings and the light fermions only transform under the more weakly coupled of the two SU(2) gauge groups. We leave a study of the effects on precision electroweak observables for future work.

6. UV completions and the resurrection of strong dynamics for EWSB

So far, we have contented ourselves with an effective theory description of our non-linear Sigma model beneath the scale $\Lambda \sim 10$ TeV, but it is interesting to contemplate a UV origin for our physics. There is a straightforward UV completion into a linear sigma model, suitably supersymmetrized to avoid quadratic divergences coming from above the scale Λ, with a supersymmetry breaking scale ~ 100 TeV. But it is fascinating that the essential features of our model can arise from strong gauge dynamics at the scale Λ. Consider, an SO(N) gauge theory with 5 Weyl Fermions Ψ_i in the fundamental representation of SO(N).

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When the SO(N) coupling becomes strong, the Fermions condense as

$$\langle \Psi_i \Psi_j \rangle \sim \Lambda^3 \Sigma_{ij} \tag{6.1}$$

breaking SU(5) $\to$ SO(5), where Λ is the SO(N) scale and the Σ_{ij} parameterize the Goldstone Bosons in an SU(5)/ SO(5) non-linear Sigma model field. Just as in our effective lagrangian description, we can now weakly gauge a subgroup of the SU(5) global symmetry, and at low energies, we have a little Higgs with an an exceptionally light mass $m_H \sim (g^2\Lambda/16\pi^2)$, even though it has large gauge coupling and a large quartic self-coupling of order g^2. Unfortunately, because Y_1, Y_2 are not orthogonal, there will be logarithmically enhanced kinetic mixing between them from a Ψ loop, which induces 1-loop quadratic divergences for the little Higgs mass from the U(1) interactions. This minor annoyance can be avoided by slightly enlarging the model. For example, 7 rather than 5 Weyl Fermions leads to an SU(7)/ SO(7) model. In this case we can weakly gauge a $G_1 \times G_2 = [\mathrm{SU}(2) \times \mathrm{U}(1)] \times \mathrm{SU}(3)$ subgroup of the SU(7) global symmetry, with the U(1) embedded as $Y = \mathrm{diag}(1, 1, -2, 0, 0, 0, 0)$. These charges satisfy our requirements: the G_1 couplings preserve an SU(4) global symmetry while the G_2 coupling preserves a different SU(4) global symmetry. There is a triplet of little Higgses in this model, consisting of a conventional Standard Model Higgs and charged singlet.

For a realistic model we still need to incorporate Fermion masses, most importantly the large top Yukawa coupling to drive electroweak symmetry breaking. Therefore an ETC-like mechanism is still likely needed to generate operators of the form of equation (3.11).

The construction of a little Higgs from strong dynamics resurrects the possibility that strong dynamics play a role in EWSB while avoiding the traditional pitfalls of technicolor, and greatly ameliorating the difficulties of extended technicolor. Technicolor and ETC are beset by three major woes: excessively large correction to precision electroweak observables, such as the S-parameter; excessively light pseudo-Goldstone bosons with electroweak quantum numbers; and large flavor-changing neutral currents. Little Higgses eliminate the first two problems: electroweak symmetry breaking is accomplished with a light weakly coupled Higgs, and there are no large corrections to the S-parameter. The pseudo-Goldstone boson issue is used to advantage: our little Higgs *is* the only excessively light pseudo-Goldstone boson with electroweak quantum numbers! As for FCNCs, the higher scale for the strong dynamics significantly relaxes the constraints, as we discuss in the next section.

7. Flavor changing neutral currents

Strictly within our low-energy effective field theory, we do not have to worry about FCNC. The only flavor-violating interactions beyond those in the SM Yukawa couplings involve the top sector, and therefore there are no large FCNCs involving the light fermions. However, in some UV completions, other, less benign spurions might be present. In order to address FCNC issues we need to speculate about the UV origin of the spurions which give the quark and lepton couplings to the Σ field.

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As stressed in [26], in those UV completions of the deconstruction models in which the light quark couplings to the sigma field arise from four Fermi interactions in the UV theory, there may also be four Fermi interactions among light quarks, with coefficients proportional to their Yukawa couplings. If the light quark four Fermi couplings are about the same size as the couplings of the light quarks to the new fermions, then for $\Lambda \lesssim 75$ TeV , some of the resulting FCNCs could exceed the experimental bounds. Also discussed in [26] were ways to ameliorate these effects, such as anomalous scaling.

Here we briefly mention some other scenarios in which one could explain the observed suppression of FCNCs. If the sigma field is a condensate of strongly interacting fermions, and so our little Higgs is composite, one might imagine that the fermions are also composite, with masses protected by approximate global symmetries [1]. It is then a simple matter to postulate global symmetries and spurions which allow the necessary Yukawa couplings while preventing excessive FCNC [27]. Another possibility is that the world is supersymmetric, with supersymmetry broken at 100 TeV or so. Then scalars with masses of 100 TeV are natural. If strongly coupled to the constituents giving rise to the nonlinear sigma fields and weakly coupled to light quarks and leptons, 100 TeV scalars could mediate interactions leading to acceptable fermion masses without excessive FCNCs [28, 29, 30].

8. Smoking guns

We will not attempt an analysis of the experimental consequences of this model here, but simply mention qualitative features of the most distinctive signals. Unlike in solutions to the hierarchy problem involving extra dimensions, deconstructed extra dimensions, or supersymmetry, the natural expectation for the mass of all new particles beyond the Standard Model is of order a TeV. Unlike in technicolor, there is a weakly coupled light Higgs and no new strong interactions below the cutoff. There are, however, several unique signatures for the discovery of new particles at the LHC. The most unique feature of the $\mathrm{SU}(5)/\mathrm{SO}(5)$ nonlinear sigma model is the ϕ electroweak triplet scalar, which will appear as three nearly degenerate scalars with charges $2, 1$ and 0. For aspects of the phenomenology see [31, 32, 33]. The other distinctive features of this model, shared with deconstruction models of electroweak symmetry breaking, are the additional $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge bosons and charge 2/3 quarks.

Without specification of the UV physics above the cutoff, the Higgs potential and couplings are determined at one loop in terms of 9 parameters of the effective theory below the cutoff — these are $g_{1,2}$, $g'_{1,2}$, $\lambda_{1,2}$, f, c and c'. Four combinations of these are determined from α, $\sin^2\theta_W$, the top mass, and the Higgs vev, while the other five could be determined from the Higgs mass, m', M'_W, M'_B, and M_ϕ, or in principle, given sufficient precise data, from a fit to electroweak observables. It is possible that the latter could give a prediction for the masses of the new particles.

We have chosen to eliminate all 1-loop quadratic divergences in the Higgs mass squared parameter. The apparent divergences in the low energy effective theory from the Standard Model gauge bosons, top quark and Higgs scalar are cancelled by new particles of similar statistics. As the masses of these new particles increase, the difference between the Standard Model 1-loop contributions and that of these new particles grows, requiring fine tuning of the Higgs mass squared parameter. Hence the masses of these new particles may be bounded by requiring that this cancellation is not finely tuned. The cutoff dependence may be absorbed into the counterterms c and c'. The logarithm might be as large as $\log(16\pi^2) \sim 5$, however to be conservative we will take it to be 1. The reasonable naturalness constraint that none of the independent contributions in (4.11), (4.12), (4.14) exceed the absolute value of the Higgs mass squared parameter by more than a factor of 10 ($\sim$ 10% fine-tuning) gives upper bounds on the new particle masses as a function of the physical Higgs mass. The new charge 2/3 quark is the most constrained:

$$m' \lesssim 2\,\mathrm{TeV}\ \left(\frac{m_H}{200\,\mathrm{GeV}}\right)^2 . \tag{8.1}$$

Two dominant decay modes are the flavor changing neutral current $T' \to Zt$, due to the mass mixing of charge 2/3 quarks with different weak charges, and $T' \to ht$. Using the expression (3.13) for the top Yukawa coupling, we conclude that $\sqrt{\lambda_1^2 + \lambda_2^2} > 2\lambda_t$. Since the mass of the heavy quark is $m' = \sqrt{\lambda_1^2 + \lambda_2^2}\, f$, the naturalness bound on m' in turn implies

$$f \lesssim 1\ \mathrm{TeV}\ \left(\frac{m_H}{200\,\mathrm{GeV}}\right)^2 . \tag{8.2}$$

Given the expectation that the couplings are all weak, (8.2) suggests that all the new particles should have masses around a few TeV and are available for an LHC discovery. However the 1-loop naturalness bounds on the new bosons are not stringent enough to be interesting. These are:

$$M'_W \lesssim 6\,\mathrm{TeV}\left(\frac{m_H}{200\,\mathrm{GeV}}\right)^2, \qquad M_\phi \lesssim 10\,\mathrm{TeV} \tag{8.3}$$

One might try to obtain tighter bounds from estimating 2-loop contributions, but these remain quadratically sensitive to the cutoff and thus constrain the cutoff physics rather than the parameters of the effective theory.

Although not guaranteed by naturalness, discovery of the new particles at the Tevatron run II is not out of the question.

9. Conclusions

Theories with a little Higgs — where the lightness of the Higgs is understood because it is a pseudo-Goldstone Boson — provide a qualitatively new framework for physics beyond the Standard Model. While the first examples of such models were inspired by deconstruction and theory space, in this paper we have seen how these ideas can be generalized to yield very economical models. The essential requirement is that the Higgs should transform nonlinearly under a collection of symmetries, which are completely broken by a collection of spurions, but no single spurion should break all the symmetries. We exploited this insight to present what we believe is the minimal possible set of new symmetries and particles needed to stabilize the weak scale against a cutoff of order $\Lambda \sim 10$TeV, without fine tuning. We have logarithmic sensitivity to the cutoff at one loop, and quadratic sensitivity at 2-loops, which is sufficient to make the electroweak symmetry breaking scale of 250 GeV natural. Our philosophy here is rather similar to that of Effective Supersymmetry [34, 35] in which only the minimal set of superpartners required for naturalness is kept lighter than the TeV scale, with all others pushed up to 10 TeV, but our particle content at the TeV scale is much more economical.

We could eliminate all UV sensitivity to some specified number of loops, and thereby obtain more predictivity, at the price of being less minimal — introducing larger symmetry groups and more particles — as was done in [14, 15]. For now, however, there is no experimental motivation to do so, as a 10 TeV cutoff is sufficient to account for the agreement of the Standard Model with precision electroweak data.

This model is the simplest example in a new class of theories of natural electroweak symmetry breaking, and clearly its phenomenology deserves further study. There are many avenues for further exploration, including generalizations beyond our minimal model, calculations of precision electroweak observables, as well as possible UV completions.

Our minimal model is remarkable in providing the first example of a theory of natural electroweak symmetry breaking with *no* new degrees of freedom beyond the Standard Model beneath a TeV. This is in sharp contrast to the MSSM, where there is no reason for the Higgs to be lighter than the superpartners. Even above a TeV, our model introduces only a very small number of new degrees of freedom that stabilize the Higgs mass. Counting all helicity states, the triplet scalar, massive gauge bosons and heavy fermion add a total of 30 new real degrees of freedom beyond the Standard Model. This is smaller than the 56 new degrees of freedom introduced in the minimal moose of our companion paper [16], and far smaller than the 126 new degrees of freedom introduced in the MSSM, not to mention the ~ 1000 new degrees of freedom introduced in theories with extra dimensions at the TeV scale. Of course, mindless minimalism is not a measure by which to judge a physical theory, but the economy of our model does illustrate the simplicity of the underlying mechanism.

Summarizing, the broad consequences of our model are threefold:

1. Electroweak symmetry breaking without fine tuning can be realized with the particle content of the minimal Standard Model below a TeV.

2. A small number of new, weakly coupled particles are required at a few TeV, including at least one heavy copy of the electroweak gauge bosons and top quark, and a scalar coupled to the Higgs.

3. The old idea of dynamical electroweak symmetry breaking can be resurrected, naturally manifesting itself at low energies as the Standard Model with a weakly coupled light Higgs.

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